Complexity growth patterns in the Big History. Preliminary results of a quantitative analysis

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Abstract: The paper presents preliminary results of a quantitative analysis of two patterns of complexity growth in the Big History – decelerating universal (cosmic) evolutionary development evidenced in the Universe for a few billions of years after the Big Bang (around 13.8 billion BP) and accelerating global (biosocial) evolutionary development observed for about 4 billion years on the planet Earth since the emergence of life on it and until the early 1970s. It is shown that the first pattern can be described with an astonishing accuracy (R^2 = 0.999996) by the following equation: $y = C_1/(t-t_1*)$, where y is the rate of the universal complexity growth (measured as a number of phase transitions [accompanied by the growth of complexity] per a unit of time), C_1 is a constant, and $t-t_1*$ is the time since the Big Bang Singularity (t_1*~13.8 billion years BP). In the meantime, it was earlier shown that the second pattern could be described with an almost as high accuracy (R^2 = 0.9989 to 0.9991) by the following equation: $y = C_2/(t_2*-t)$, where y is the rate of accelerating global (biosocial) evolutionary development, C_2 is another constant, and t_2*-t is the time till the 21st century Singularity (t_2*, estimated to be around 2027, or 2029 CE). Thus, the post-Big-Bang hyperbolic decrease of universal complexity growth rate and the hyperbolic increase of the growth rate of global complexity in the last 4 billion years proceeded following the same law. We are dealing here with a perfect symmetry: (1) the rate of the universal (cosmic) complexity growth decreases when we move from the Big Bang Singularity, whereas the rate of the global complexity growth increase when we approach the 21st century Singularity; (2) more specifically, as the time since the Big Bang Singularity increases n times, the universal (cosmic) complexity growth rate decreases the same n times, whereas when the time till the 21st century Singularity decreased n times, the global complexity growth rate increased the same n times. A somehow more complex symmetry is observed as regards the interaction between energy dynamics and complexity growth within both processes. The implications of the symmetry of both patterns are discussed.

1. Introduction

The point that within the Big History the decelerating growth of complexity in the Universe observed after the Big Bang can be contrasted with the accelerating growth of complexity traced on our planet for four billion years after the emergence of life on the Earth has been already noticed on quite a number of occasions (e.g., Panov, 2007, 2008; Tsirel 2009; LePoire, 2014, 2016, 2020b; Nazaretyan 2017b; Panov et al., 2020; Faixat, 2022).

However, till now nobody seems to have undertaken a detailed mathematical comparison of these patterns. In fact, by now a rather thorough mathematical analysis has been only performed as regards the global pattern of accelerating evolution.[1]

David LePoire appears to be the only person to have conducted some mathematical comparison of the two abovementioned patterns (LePoire, 2014, see Fig. 1). However, this has been only published as a presentation at the 2nd International Big History Association Conference at Dominican College in San Rafael, CA in August 2014 (LePoire, 2014) and it does not appear to have been noticed by big historians. In addition, in this presentation, his analysis of the post-Big Bang universal evolution deceleration pattern, while being basically correct, was rather brief and lacked much detail (unlike his later very thorough and detailed mathematical analysis of the

[1] See von Foerster et al., 1960; Hoerner, 1975; Taagepera, 1976, 1979; Jantsch, 1980; Kapitza, 1992, 1996a, 1996b, 1999, 2003, 2006, 2007, 2010; Kremer, 1993; Johansen & Sornette, 2001; Kurzweil, 2001, 2005; Modis, 2002, 2003, 2020; Tsirel, 2004; Korotayev, 2005, 2006a, 2006b, 2007, 2013, 2018, 2020a, 2020b; Panov, 2004, 2005, 2011, 2017, 2020; Grinchenko, 2006; Korotayev & Khaltourina, 2006; Korotayev et al., 2006a, 2006b, 2015, 2016; Grinin, 2006; Markov & Korotayev, 2007, 2008; Grinin & Korotayev, 2009, 2015; Grinchenko & Shchapova, 2010, 2020; Markov et al., 2010; Korotayev & S. Malkov, 2012; Grinin et al., 2013, 2014, 2015, 2020a, 2020b; Korotayev & Grinin, 2013; Korotayev & Markov, 2014, 2015; LePoire, 2014, 2015a, 2015b, 2016, 2020a, 2020b; Korotayev & A. Malkov, 2016; Korotayev & Zinkina, 2017; Podlazov, 2017; Dobrolyubov, 2020; Fomin, 2020; Malkov, 2020; Widdowson, 2020; Faixat, 2022).

accelerating growth of complexity traced on our planet for four billion years after the emergence of life on the Earth [LePoire, 2016, 2020a, 2020b]).

This paper aims at filling this gap by providing a more detailed quantitative analysis of the two abovementioned complexity growth patterns in the Big History. In addition, my comparative analysis is in no way a repetition of the one performed by David LePoire in 2014, as the comparative methodology I apply is quite different from LePoire's. Thus, my analysis does not contradict his, but rather complements it.

This article is structured as follows. In its first part, I present a summary of my previous systematic quantitative analysis of the accelerating global (biosocial) complexity growth observed for about 4 billion years on our planet. In the second part, I apply the same methodology that I have applied to analyze this accelerating pattern to the analysis of decelerating universal complexity growth evidenced in the Universe for a few billions of years since the Big Bang Singularity (around 13.8 billion years BP). Finally, the third part offers a systematic comparison of the both patterns.

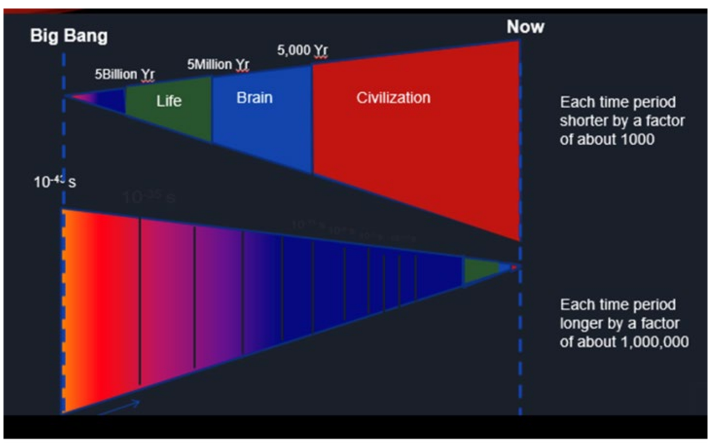

Fig. 1 Two Contrasting Views of Time Scales. *Source*: LePoire, 2014

2 Summary of Previous Results of the Accelerating Complexity Growth on Our Planet

Raymond Kurzweil was one of the first to arrange the major evolutionary shifts of a very significant part of the Big History along the hyperbolic curve that can be described by an equation with a mathematical singularity. For example, at page 18 of his bestseller *The Singularity is Near* (2006) the time sequence is shown[2]. However, rather surprisingly, Kurzweil does not appear to have recognized that the curve represented at this figure is hyperbolic, and that it is described by an equation possessing a true mathematical singularity (what is more the value of this singularity, 2029 is not so far from the one professed by Kurzweil himself [see Ranj 2016]).

A very important contribution to the quantitative analysis of the accelerating growth of complexity traced on our planet for four billion years was done in 2003 by a physicist from Lomonosov Moscow State University Alexander Panov[3]. Panov analyzed an essentially similar time series taken from entirely different sources but arrived at very similar conclusions, but in a much more advanced form. It is very important that he made a step (to which Kurzweil was very close but which he did not make actually) that allows to make the analysis of the time series in question much more transparent.

In his 2005 book Kurweil plotted at the Y-axis of his diagrams "time to next event", which hindered for him their interpretation in a rather significant way. In his 2001 essay at page 5 while analyzing a diagram with a similar time series (whose source, incidentally, was not indicated), Kurzweil began speaking about the acceleration of "paradigm shift rate" (Kurzweil 2001: 5), but almost immediately switched to another theme. However, what was necessary to make his diagrams much more intelligible was to plot at Y-axis not "Time to Next Event", but just "Paradigm Shift Rate" – precisely as was done by Panov. Indeed, to transform the time to next paradigm shift into paradigm shift rate one needed to do a rather simple thing – to take one year and to divide it by time to next paradigm shift; this will yield number of paradigm shifts per year, that is just a "Paradigm Shift Rate". As we have already said, this was not done by Kurzweil but was done by Panov who obtained the following graphs as a result (see Fig. 2):

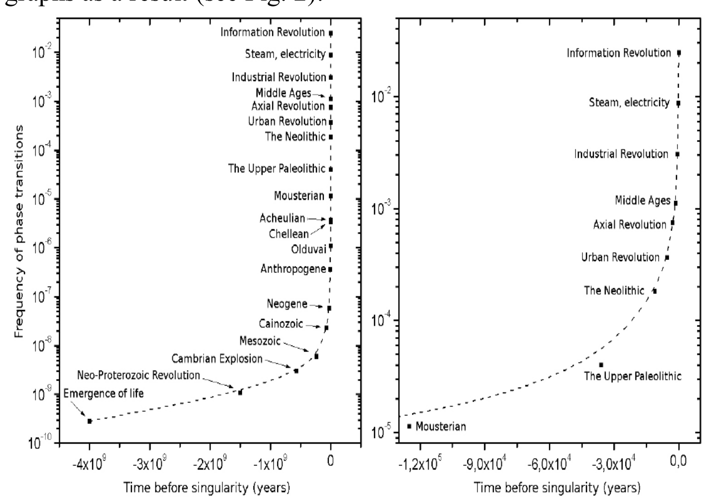

Fig. 2 Dynamics of the global complexity growth rate according to Panov. *Source* Nazaretyan 2018: 31, Fig. 3. The left-hand diagram depicts the acceleration of the global complexity growth rate starting from 4 billion years BP, whereas the right-hand diagram describes this for the human part of the Big History.

[2] Actually, a prototype of this figure (but in a double logarithmic scale) was reproduced by Kurzweil already in 2001 in his essay "The Law of Accelerating Returns" at page 5.

[3] His calculations described below were first presented in November 2003 at the Academic Seminar of the State Astronomic Institute in Moscow (Nazaretyan 2005: 69) and subsequently published in his articles (Panov 2004, 2005, 2011, 2017, 2020) and monograph (Panov 2008).

The mathematical interpretation of Panov's graph is much easier and more straightforward. Note that Panov himself denoted the variable plotted at Y-axis as "Frequency of the phase transitions per year". However, it is quite clear that Panov's "phase transition" is a synonym of Kurzweil's "paradigm shift", whereas "frequency of the phase transitions per year" describes just "paradigm shift rate", or global evolutionary macrodevelopment rate. This transformation makes it much easier to detect rigorously the pattern of acceleration of the global complexity growth rate.[4]

This was compared with the sequence presented in a paper by Theodore Modis "The Limits of Complexity and Change" (2003) prepared in its turn on the basis of his earlier article published in the *Technological Forecasting and Social Change* (2002) (note that in this article Modis denotes "phase transitions" as "complexity jumps"). Fortunately, Modis provided all the necessary dates in his articles, which made it perfectly possible to analyze this time series mathematically.

At the next step I let the X-axis represent the time before the singularity (whereas the Y-axis represented the macrodevelopment rate) – and calculated the singularity date by getting such a power-law curve that would describe our time series in the most accurate way. The results of this analysis are presented in Fig. 3 (note that our mathematical analysis identified the Singularity date for this time series as 2029 CE).

As we see, our power-law regression of the last "Countdown to Singularity" time series identifies the following best fit equation describing this time series in an almost ideally accurate ($R^2 = 0.999$) way:

$$y = \frac{2.054}{x^{1.003}}, \tag{1}$$

where y is the global macrodevelopment rate, x is the time remaining till the singularity, and 2.054 and 1.003 are constants. Note that the denominator's exponent (1.003) turns out to be only negligibly different from 1 (well within the error margins); Of course, x (the time remaining till the singularity) at the moment of time t equals $t^* - t$, where t^* is the time of singularity. Finally, let us recollect that our power-law analysis of the transformed Modis – Kurzweil series has identified the singularity date as 2029 CE. Thus, Eq. (1) can be further re-written in the following way:

$$y_t = \frac{2.054}{2029 - t}. \tag{2}$$

Now, let us apply a similar methodology to analyze mathematically the series of global macroevolutionary "phase transitions"/ "biospheric revolutions" compiled by Alexander Panov (2005; see also Panov 2008, 2011, 2017).

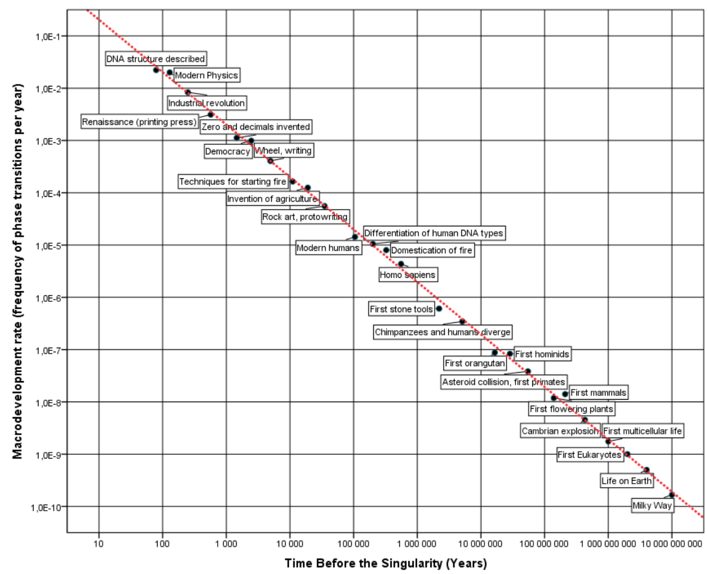

Fig. 3 Scatterplot of the phase transition points from the Modis – Kurzweil list with the fitted power-law regression line (double logarithmic scale) – for the Singularity date identified as 2029 CE with the least squares method

Note that Alexander Panov and Theodore Modis compiled their time series entirely independently of each other. As suggest my personal communications with both Panov and Modis, none of them knew that at almost the same time[5] in another part of Europe another person compiled a similar time series (Alexander Panov worked in Moscow, whereas Theodore Modis worked in Geneva). They relied on entirely different sources and the resultant time series turned out to be very far from being identical (see, e.g., Table 1).

As one can see for a major part of the planetary history (between the Cambrian explosion and the formation of *Homo sapiens sapiens*) the correlation between the two series is really weak; they look as really independent (and rather different) series.

It appears appropriate to recollect at this point that in their famous article published in the journal *Science* in 1960 von Foerster, Mora, and Amiot presented their results of the analysis of the world population growth pattern. They showed that between 1 and 1958 CE the world's population (N) dynamics can be described in an extremely accurate way with the following astonishingly simple equation:

$$N_t = \frac{C}{(t^* - t)^{0.99}}, \tag{3}$$

where N_t is the world population at time t, and C and t^* are constants, with t^* corresponding to the so called "demographic

[4] Note, however, that most of the students of the global accelerating growth of complexity still prefer to deal with periods between phase transitions rather than phase transition rates (see, e.g., Panov, 2005, 2020; Grinchenko & Shchapova, 2010, 2020; LePoire, 2014, 2016, 2020a, 2020b; Dobrolyubov, 2020; Malkov, 2020; Faixat, 2022).

[5] Modis first presented his results in an article in *Technological Forecasting and Social Change* (that Panov only read in March 2018 after it was sent to him by me) in 2002, whereas Panov first presented his results next year at the Academic Seminar of the State Astronomic Institute in Moscow.

Modis – Kurzweil series	Panov (2005) series
(6) First mammals, first birds, first dinosaurs – *210 million years ago*. **(7) First flowering plants**, oldest angiosperm fossil – *139 million years ago*. **(8) First primates**/ asteroid collision/ mass extinction (including dinosaurs) – *54.6 million years ago*. **(9) First hominids**, first humanoids – *28.5 million years ago*. **(10) First orangutan**, origin of proconsul – *16.5 million years ago*. **(11) Chimpanzees and humans diverge**, earliest hominid bipedalism – *5.1 million years ago*. **(12) First stone tools**, first humans, *Homo erectus – 2.2 million years ago*. **(13) Emergence of *Homo sapiens*** – *555,000 years ago*. **(14) Domestication of fire** / *Homo heidelbergensis – 325,000 years ago*. **(15) Differentiation of human DNA types** – *200,000 years ago*.	**(3) Reptiles revolution** (The beginning of Mesozoic era) – *235 million years ago*. **(4) Mammalia revolution** (The beginning of the Cenozoic era). Dinosaurs died out. Mammalia animals became the leader of the evolution on the terra firma. – *66 million years ago*.· **(5) Hominoid revolution** (The beginning of the Neogene period). A big evolution explosion of Hominoidae (apes) – *22.5 million years ago*. **(6) The beginning of Quaternary period (Anthropogene) /** The first primitive Homo genus (hominidae) separated from hominoidae – *4.4 million years ago*. **(7) Palaeolithic revolution /** Homo habilis, the first stone implements – *1.8 million years ago*. **(8) The beginning of Chelles period** – *650,000 years ago*. Fire, Homo erectus. **(9) The beginning of Acheulean period**. Standardized symmetric stone implements.– *400,000 years ago*.

Table 1 Correlation between the phase transition lists of Modis and Panov for the period between 400 million years ago and 150,000 years ago

singularity". Parameter t^* was estimated by von Foerster and his colleagues as 2026.87, which corresponds to November 13, 2026; this made it possible for them to supply their article with a public-relations masterpiece title – "Doomsday: Friday, 13 November, A.D. 2026" (von Foerster, Mora, Amiot 1960). Note that von Foerster and his colleagues detected the hyperbolic pattern of world population growth for 1 CE –1958 CE; later it was shown that this pattern continued for a few years after 1958, and also that it can be traced for many millennia BCE (Kapitza 1996a, 1996b, 1999; Kremer 1993; Tsirel 2004; Podlazov 2000, 2001, 2002; Korotayev, Malkov, Khaltourina 2006a, 2006b). In fact, Kremer (1993) claims that this pattern is traced since 1 000 000 BP, whereas Kapitza (1996a, 1996b, 2003, 2006, 2010) even insists that it can be found since 4 000 000 BP.

It is difficult not to see that the world population growth acceleration pattern detected by von Foerster in the empirical data on the world population dynamics between 1 and 1958 turns out to be virtually identical with the one that has been detected above with respect to both Modis – Kurzweil and Panov series describing the planetary macroevolutionary development acceleration. Note that the power-law regression has yielded for all the three series the value of exponent β being extremely close to 1 (1.003 for the Modis – Kurzweil series, 1.01 for Panov, and 0.99 for von Foerster).

However, the resultant proximity of parameter t^* (that is just the singularity time point) estimates is also really impressive (the power-law regression suggests 2029 for the Modis – Kurzweil series, 2027 for Panov series, and just the same 2027 for von Foerster series[6]).

We have already mentioned that, as was the case with equations (1) and (2) above, in von Foerster's Eq. (3) the denominator's exponent (0.99) turns out to be only negligibly different from 1, and as was already suggested by von Hoerner (1975) and Kapitza (1992, 1999). As we see the resultant equation turns out to be entirely identical with Eq. (2) above that described so accurately the overall planetary macrodevelopment acceleration pattern since at list 4 billion years ago. Note that Eq. (3) has turned out to be as capable to describe in an extremely accurate way the world population dynamics (up to the early 1970s), as Eq. (2) is capable to describe the overall pattern of macredevopment acceleration (at least between 4 billion BCE and the present). We will show just an example of such a fit.

Let us take Eq. (3). Now replace t^* with 2027 (that is the result

[6] Note that the power-law regression that produced this value for the world populations series had been performed more than 50 years before a similar regression produced the same value of t^* for the Panov series (actually, the first regression was performed before the birth of the author of the present article). Still I would not take too seriously such astonishingly similar values of t^* parameter produced by different power-law regressions for very different time series in very different years; of course, there is a very high degree of coincidence here. In any case, as we will see below, there are no grounds at all to expect anything like Doomsday on Friday, November 13, A.D. 2026…

of just rounding of von Foester's number, 2026.87), and replace C with 215000.[7] This gives us a version of von Foerster – von Hoerner – Kapitza Eq. (4) with certain parameters:

$$N_t = \frac{215000}{2027 - t}. \tag{4}$$

The overall correlation between the curve generated by von Foerster's equation and the most detailed series of empirical estimates looks as follows (see Fig. 4):

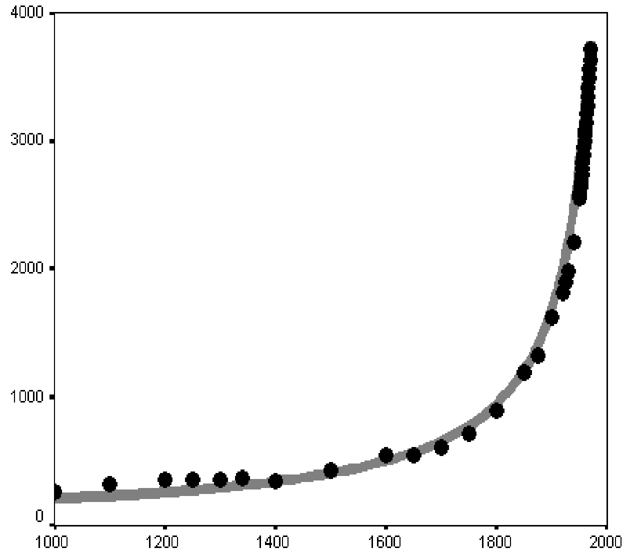

Fig. 4 Correlation between Empirical Estimates of World Population (in millions, 1000 – 1970) and the curve generated by von Foerster's Equation (3). *Note:* black markers correspond to empirical estimates of the world population by McEvedy and Jones (1978) for 1000–1950 and UN Population Division (2022) for 1950–1970. The grey curve has been generated by von Foerster's Eq. (10). $R^2 = 0.996$

As we see, indeed, Eq. (4) has turned out to be as capable to describe in an extremely accurate way the world population dynamics (up to the early 1970s), as Eq. (2) is capable to describe the overall pattern of global complexity growth rate acceleration.

We have shown that that the fact that, up to the beginning of the 1970s, the world population size (N) and the global complexity increase rate (y) in the Panov series grew following the same law ($x_t = C / 2027 - t$), is by no means a coincidence; it is rather a manifestation of a fairly deep pattern of the global evolution. Thus, at the social phase of universal and global history, the hyperbolic growth of the rate of increase in global complexity and the hyperbolic growth of the Earth's population are two closely related aspects of a single process. We have demonstrated that Eq. (4) can be derived from Eq. (2) and the other way around (e.g., Korotayev, 2020a, 2020b).

I must say that I had serious doubts when I first got across calculations of Panov and Modis (and I am not surprised that most historians get very similar doubts when they see their works). I had many complaints regarding the accuracy of many of their descriptions of their "canonical milestones", their selection, and their datings (see, e.g., Korotayev 2015). I have only started taking their calculations seriously, when I analyzed myself the two respective time series compiled (as we have seen above) entirely independently by two independently working scientists using entirely different sources with a mathematical model not applied to their analysis either by Modis or by Panov, and found out that they are described in an extremely accurate way by an almost identical mathematical hyperbolic function – suggesting the actual presence of a rather simple hyperbolic planetary macroevolution acceleration pattern observed on the Earth for the last 4 billion years. This impression became even stronger when the equation describing the planetary macroevolution acceleration pattern turned out to be identical with the equation that was found by Heinz von Foerster in 1960 to describe in an extremely accurate way the global population growth acceleration pattern between 1 and 1958 CE.

But how seriously should we take the prediction of "singularity" contained in such mathematical models? Should we really expect with Kurzweil that around 2029 we should deal with a few orders of magnitude acceleration of the technological growth (indeed, predicted by Eq. (2) if we take it literally[8])?

I do not think so. This is suggested, for example, by the empirical data on the world population dynamics. As we remember, the global population growth acceleration pattern discovered by Heinz von Foerster is identical with planetary macroevolutionary acceleration patterns of Modis – Kurzweil and Panov, and it is characterized by the singularity parameter (2027 CE) that is simply identical for Panov and has just 2 year difference with Modis – Kurzweil. However, what are the grounds to expect that by Friday, November 13, A.D. 2026 the world population growth rate will increase by a few orders of magnitude as is implied by von Foerster equation? The answer to this question is very clear. There are no grounds to expect this at all. Indeed, as we showed quite time ago, "von Foerster and his colleagues did not imply that the world population on [November 13, A.D. 2026] could actually become infinite. The real implication was that the world population growth pattern that was followed for many centuries prior to 1960 was about to come to an end and be transformed into a radically different pattern. Note that this prediction began to be fulfilled only in a few years after the "Doomsday" paper was published" (Korotayev 2007: 154).

Indeed, starting from the early 1970s the world population growth curve began to diverge more and more from the almost ideal hyperbolic shape it had before (see Fig. 4) (see, e.g., Kapitza, 2003, 2006, 2007, 2010; Livi-Bacci 2012; Korotayev, Malkov, Khaltourina 2006a, 2006b; Korotayev, Goldstone, Zinkina 2015; Grinin, Korotayev 2015; UN Population Division 2022), and in recent decades it has been taken more and more clearly logistic shape – the trend towards hyperbolic acceleration has been clearly

[7] Note that all the calculations below of the world population are conducted in millions. Note also that the value of parameter C used by us is a bit different from the one used by von Foerster.

[8] This is done, for example, by Nazaretyan (2017a, 2017b, 2018, 2020).

replaced with the logistic slow-down, with a clear perspective of transition to a negative population growth rate (see Fig. 5):

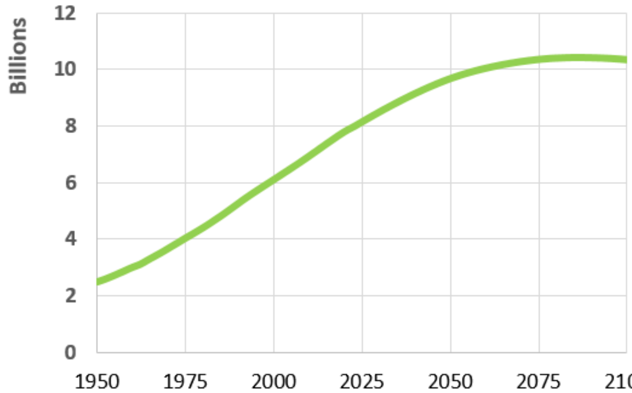

Fig. 5 World population dynamics (billions), empirical estimates of the UN Population Division for 1950–2015 with its middle forecast to 2100. *Data source*: UN Population Division 2022

In some respect, it may be said that von Foerster did discover the singularity of the human demographic history; it may be said that he detected that the human World System was approaching the singular period in its history when the hyperbolic accelerating trend that it had been following for a few millennia (and even a few millions of years according to some) would be replaced with an opposite decelerating trend. The process of this trend reversal has been studied very thoroughly by now (see, e.g., Vishnevsky 1976, 2005; Chesnais 1992; Caldwell et al. 2006; Khaltourina & Korotayev, 2007; Korotayev, Malkov, Khaltourina 2006a, 2006b; Korotayev 2009; Gould 2009; Dyson 2010; Reher 2011; Livi-Bacci, 2012; Choi, 2016; Podlazov, 2017) and is known as the "global demographic transition" (Kapitza 1999, 2003, 2006, 2010; Podlazov 2017). Note that in case of global demographic evolution the transition from the hyperbolic acceleration to logistic deceleration started a few decades before the singularity point mathematically detected by von Foerster.

There are all grounds to maintain that the deceleration of planetary macroevolutionary development has also already begun – and it started a few decades before the singularity time points detected both in Modis – Kurzweil and Panov. This is well supported by the growing body of evidence suggesting the start of the long term deceleration of the global techo-scientific and economic growth rates in the recent decades (see, e.g., Krylov 1999, 2002, 2007; Huebner 2005; Khaltourina & Korotayev, 2007; Maddison 2007; Korotayev and Bogevolnov 2010; Korotayev et al. 2010; Modis 2002, 2005, 2012, 2020; Akaev 2010; Gordon 2012; Teulings & Baldwin, 2014; Piketty 2014; LePoire 2005, 2009, 2013, 2015a, 2015b, 2020a, 2020b; Korotayev & Bilyuga 2016; Popović, 2018; LePoire & Chandrankunnel, 2020; LePoire & Devezas, 2020; Widdowson, 2020).

Now, let us sum up our quantitative analysis of the accelerating growth of complexity traced on our planet for four billion years since the emergence of life on the Earth.

It may be said that the general formula of the acceleration of the global complexity growth

$$y = \frac{C}{t^{*} - t} \tag{5}$$

can be described as follows:

- The rate of the global complexity growth increases when we approach the Singularity.
- As the time until the Singularity decreases *n* times, the global complexity growth rate increases the same *n* times.
- Thus, if the time until the Singularity lessens by a factor of 3, the speed of the global complexity growth rises 3 times; if the time till the Singularity diminishes 10 times, the global complexity growth rate escalates by a factor of 10, and so on.

Let us apply now the same methodology that we have applied earlier to analyze the abovementioned accelerating pattern to the analysis of decelerating universal (cosmic) evolutionary development evidenced in the Universe for a few billions of years since the Big Bang Singularity.

3 Decelerating Universal (Cosmic) Evolutionary Development After the Big Bang

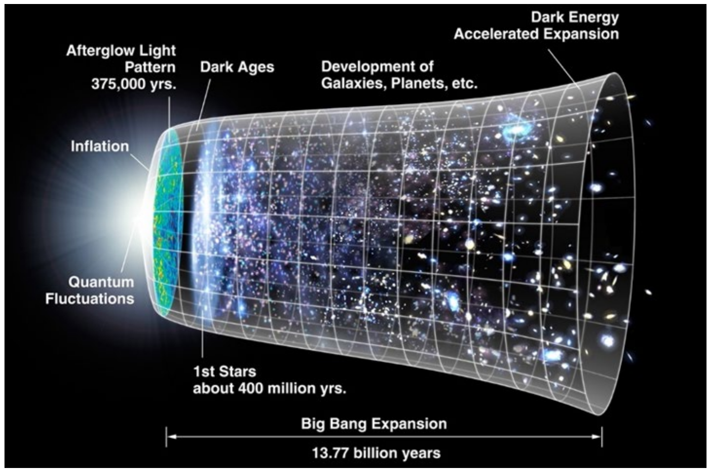

Fig. 6 Timeline of the universe. A representation of the evolution of the universe over 13.77 billion years. Source: https://en.wikipedia.org/wiki/Big_History#/media/File:CMB_Timeline300_no_WMAP.jpg

We have used the following time series for our analysis (shown in Table 2), taking into account the following phases of the universal complexity growth. The major phase transitions and phases of complexity growth in the Universe, as well as their dating in Notes. *Data sources for Tables 2*: Baumann, 2022; Chaisson, 2001; Coc, 2017; Coc et al., 2014; Gorbunov & Rubakov, 2018; Hawking, 2009; Karki, 2010; Loeb, 2006; May et al., 2008; Morison, 2015;

Mukhanov, 2005; Panov, 2008; Petter, 2013; Ryden, 2017; Spier, 2010; Sunayev & Chuba, 2009. Note that the list of phase transitions above does not include the transition from the radiation-dominated era to the matter-dominated one around 47 thousand years after the Singularity and the transition from the matter-dominated era to the dark-energy-dominated one [accompanied by the acceleration of the expansion of the Universe] around 9.8 billion years after the Singularity, as both of these important milestones of the cosmic history do not appear to have been accompanied by any clear increase in complexity. However, it is important to emphasize that our additional tests have indicated that their inclusion does not affect the results of our calculations in any significant way.

To identify an equation describing the post-Big-Bang decelerating growth of the complexity in the Universe we apply to the table above the same type of mathematical analysis that we applied earlier to the time series of Modis – Kurzweil and Panov. Thus, we correlate the frequency of phase transitions in the given Big History epoch with the time period separating this epoch from the Big Bang Singularity (see columns 3 and 5 in Table 2; the values used for calculations whose results are presented in Fig. 7 are highlighted with a bold font in columns 3 and 5 of Table 2).

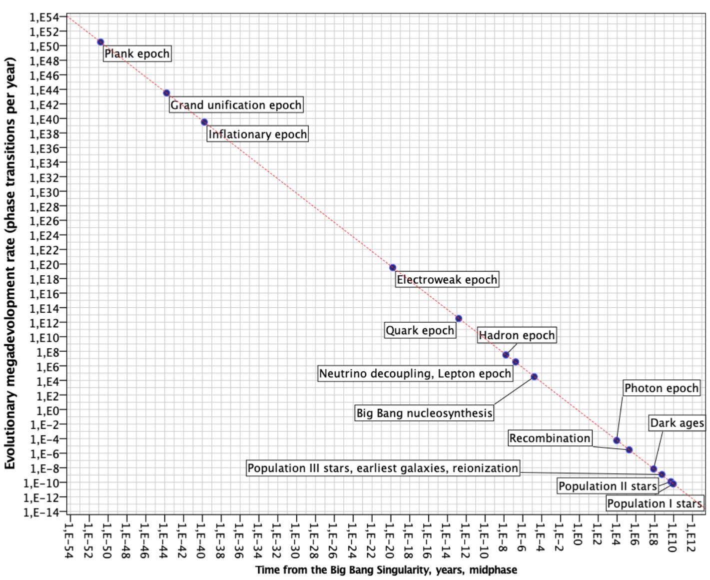

Fig. 7 Correlation between the time since the Big Bang Singularity and universal evolutionary megadevelopment rate (phase transitions per year). Scatterplot of the phases of the growth of complexity in the Universe, with the fitted power-law regression line (log-log scale)

As we see, our power-law regression of the time series of phase transitions of the post-Bing-Bang-Singularity complexity growth in the Universe outlined in Table 2 has identified the following best fit equation describing this time series:

$$y = \frac{0.549}{x^{0.998}}, \tag{6}$$

where y is the universal evolutionary megadevolopment rate (phase transitions per year), x is the time elapsed since the Big Bang Singularity, and 0.549 and 0.998 are constants. Note that the fit between the theoretical curve generated by simple power-law Eq. (13) and the empirical estimates of the complexity growth deceleration dynamics in the Universe spelled out in Table 2 (R^2 = 0.999996) has turned to be even higher than we observed above with respect to very similar power-law equations describing the global complexity growth acceleration pattern as regards Modis – Kurzweil series (Eq. (1); $R^2 = 0.9989$) and Panov series (Eq. (2); $R^2 = 0.9991$). Note that the difference of the denominator's exponent from 1 (0.998 – 1 = – 0.002) turns out to be as negligible as we could see it above with Eq. (1) describing the Modis – Kurzweil series (1.003 – 1 = 0.003) and Eq. (2) describing the Panov series (1.01 – 1 = 0.01). Hence, as we have seen this above as regards Eqs. (1) and (2), there are all grounds to use this equation in the following simplified form:

$$y = \frac{0.549}{x}, \tag{7}$$

where y is the universal complexity growth rate (phase transitions per year), x is the time elapsed since the Big Bang Singularity, and 0.549 is a constant.

However, the correlation seems too good. In fact, this type of correlation follows from the type of data and the definition of the complexity rate. The data has large relative differences in time such that the difference between the time of an event and its predecessor, at a much earlier time, is just the time of the event. When complexity is defined as the reciprocal of this time difference, the curve is effectively being defined such that C(t)=A/t, independent of the data. So this does not seem to be a good test for a singularity trend.

A different formulation of a singularity is that equally weighted events would occur with a geometric sequence in time from (or toward) the singularity time. For the Panov and Modis sequences of the biosocial evolution on earth this factor is about a third. This would mean that the next event occurs at about 1/3 of the time before the singularity time. So, an event occurring at 1,500 years before the singularity time would be expected to be followed by an event at 500 years (1500/3) before the singularity time, followed by the next event at 167 years before the singularity time. With this fractal sequence there would be an infinite number of events before the project singularity time. Of course this would never happen in a real physical sequence. This can be analyzed by placing the events sequentially and using the event number to perform a correlation. A true geometric sequence of events would have the same factor of increased time until the next event. This plot is shown below (Fig. 8), where the 10 events give an R-Square

Phases of the universal complexity growth	$t - t^*$ (**seconds** since the Big Bang Singularity)	$t - t^*$ (**years** since the Big Bang Singularity)	*Time between phases (years)*	*Universal complexity growth rate (phase transitions per year)*	Radiation energy of the Universe, in electronvolts (eV)	Radiation energy (temperature) of the Universe, in Kelvins (K)
Planck epoch starts	10^{-47}	$3.17*10^{-55}$				
Planck epoch mid-phase	$5*10^{-44}$	**$1.58*10^{-51}$**	$3.17*10^{-51}$	**$3.16*10^{50}$**	**10^{28}**	$1.16*10^{32}$
Planck epoch > Grand unification epoch	10^{-43}	$3.17*10^{-51}$				
Grand unification epoch mid-phase	$5*10^{-37}$	**$1.58*10^{-44}$**	$3.17*10^{-44}$	**$3.16*10^{43}$**	**10^{25}**	$1.16*10^{29}$
Grand unification epoch > Inflationary epoch	10^{-36}	$3.17*10^{-44}$				
Inflationary epoch mid-phase	$5*10^{-33}$	**$1.58*10^{-40}$**	$3.17*10^{-40}$	**$3.16*10^{39}$**	**$5*10^{23}$**	$5.8*10^{27}$
Inflationary epoch > Electroweak epoch	10^{-32}	$3.17*10^{-40}$				
Electroweak epoch mid-phase	$5*10^{-13}$	**$1.58*10^{-20}$**	$3.17*10^{-20}$	**$3.16*10^{19}$**	**150 billion eV (150 GeV)**	$1.74*10^{15}$
Electroweak epoch > Quark epoch	10^{-12} (one trillionth of a second)	$3.17*10^{-20}$				
Quark epoch mid-phase	$5*10^{-06}$	**$1.58*10^{-13}$**	$3.17*10^{-13}$ of a year (~1 millionth of a second)	**$3.16*10^{12}$ (3.16 trillion phase transitions per year)**	**75.1 billion eV (75.1 GeV)**	$8.71*10^{14}$ (871 trillion K)
Quark epoch > Hadron epoch	10^{-05} (0.00001, 10 millionths of a second)	$3.17*10^{-13}$				
Hadron epoch mid-phase	0.500005	**$1.58*10^{-8}$**	$3.17*10^{-8}$ of a year (~1 second)	**$3.16*10^{7}$ (31.6 million phase transitions per year)**	**75.5 million eV (75.5 MeV)**	$8.76*10^{11}$ (876 billion K)
Hadron epoch > Lepton epoch	1 second since the Big Bang Singularity	$3.17*10^{-8}$				
Lepton epoch, Neutrino decoupling, mid-phase	5.5 seconds	**$1.74*10^{-7}$**	$2.87*10^{-7}$ of a year (~9 seconds)	**$3.51*10^{6}$ (3.51 million phase transitions per year)**	**550,000 (550 KeV)**	$6.38*10^{9}$ (6.38 billion K)
Lepton epoch > Big Bang nucleosynthesis	10 seconds	$3.17*10^{-7}$				
Big Bang nucleosynthesis mid-phase	505 seconds	**$1.60*10^{-5}$**	$3.14*10^{-5}$	**$3.19*10^{4}$ (31,900 phase transitions per year)**	**50,500 (50.5 KeV)**	$5.86*10^{8}$ (586 million K)
Big Bang nucleosynthesis > Photon epoch	1000 seconds	$3.17*10^{-5}$				

Phases of the universal complexity growth	*t – t** (**seconds** since the Big Bang Singularity)	*t – t** (**years** since the Big Bang Singularity)	*Time between phases (years)*	*Universal complexity growth rate (phase transitions per year)*	Radiation energy of the Universe, in electronvolts (eV)	Radiation energy (temperature) of the Universe, in Kelvins (K)
Photon epoch mid-phase	$2.84*10^{11}$	**$9.0*10^{3}$ (9 thousand years since the B. Bang Singularity)**	$1.8*10^{4}$ (18 thousand years)	**$5.56*10^{-5}$ (5.56 phase transitions per 100 thousand years)**	**500 eV**	$5.86*10^{6}$ (5.86 million K)
Photon epoch > Recombination	$5.68*10^{11}$	$1.8*10^{4}$ (18 thousand years)				
Recombination mid-phase	$6.12*10^{12}$	**194 thousand years AS**	$3.52*10^{5}$ (352 thousand years)	**$2.84*10^{-6}$ (2.28 phase transitions per 1 million years)**	**1 eV**	$1.16*10^{4}$ (11.6 thousand K)
Recombination > Dark ages	$1.17*10^{13}$	370 thousand years since the B. Bang Singularity				
Dark ages mid-phase	$2.37*10^{15}$	**75.2 million (13.7 billion years BP)**	$1.496*10^{8}$ (149.63 million years)	**$6.68*10^{-9}$ (6.68 phase transitions per 1 billion years)**	**0.203 eV**	2,350 K
Dark ages > Population III stars	$4.73*10^{15}$	150 million (13.625 billion years BP)				
Population III stars, earliest galaxies, reionization, mid-phase	$1.81*10^{16}$	**575 million (13.2 billion years BP)**	$8.5*10^{8}$ (850 million years)	**$1.18*10^{-9}$ (1.18 phase transitions per 1 billion years)**	**0.0034 eV**	39.5 K
Population III stars > 2nd generation of stars	$3.16*10^{16}$	1 billion (12 billion years BP)				
First 3rd generation stars appear against the background of predominance of the 2nd generation of stars, medium complexity galaxies, primitive planets, primitive chemical evolution, mid-phase	$1.61*10^{17}$	**5.1 billion (8,7 billion years BP)**	8.20E+09 $8.2*10^{9}$ (8.2 billion years)	**$1.22*10^{-10}$ (1.22 phase transitions per 10 billion years)**	**$1.89*10^{3}$ eV**	22 K
Predominance of the 2nd population of stars > predominance of the 3rd generation of stars	$2.90*10^{17}$	9.2 billion (4.6 billion years BP)				
Predominance of the 3rd generation of stars, complex galaxies, complex planets, complex chemical evolution	After $2.90*10^{17}$	After 9.2 billion years AS (after 4.6 billion years BP)	?	?	**$3.79*10^{-4}$ eV**	4.4 K

Table 2 Phase transitions and phases of the complexity growth in the Universe (advanced version)

of 0.97 (in log transformed data) with a best fit of 7.6 decades for the time scaling factor (i.e., $4x10^7$).

A factor of about 6 decades in time would be expected if the energy scaled by 1,000 (due to the relationship of temperature and time after the big bang). A factor of 1000 in energy phenomena is seen in the middle range of physics phenomena from the proton mass 1,000 MeV, the electron pair production mass and typical nuclear excitation energy of 1 MeV, the ionization energy of elements at around 1 keV, a chemical binding energy around 1 eV, and intermolecular binding energies in the meV range (room temperature e = 25 meV). While this range of energy scales allows for separation of phenomena by temperature, it is not fundamentally known why it is that way.

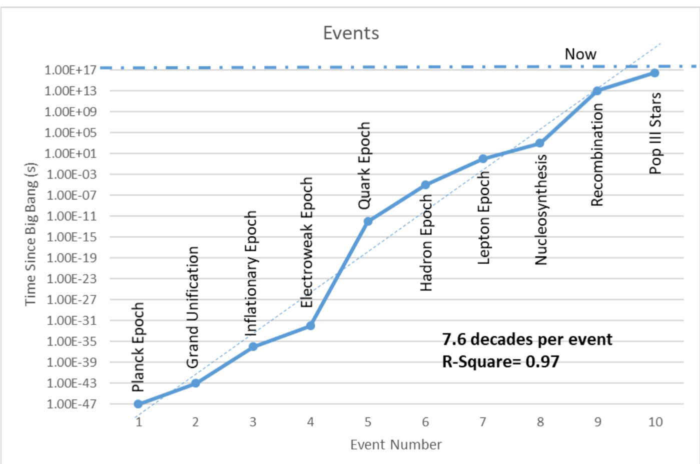

Fig. 8. Geometric sequence of post-Big-Singularity complexity jumps

Thus, our analysis has demonstrated that the decelerating universal (cosmic) evolutionary development evidenced in the Universe for a few billions of years since the Big Bang Singularity can be very accurately described by the following equation:

$$y_t = \frac{C_2}{t-t^*} \qquad (8)$$

where y_t is the rate of the universal complexity growth (complexity jumps per a unit of time) at time t; t^* is the time of the Bing Bang singularity, and C_2 is a constant.

Compare now this decelerating pattern of the universal (cosmic) evolutionary development evidenced in the Universe for a few billions of years since the Big Bang Singularity with the accelerating pattern of complexity growth traced on our planet for four billion years since the emergence of life on the Earth detected in the series of Modis – Kurzweil and Panov (see Fig. 3 and Table 1): This comparison may be also summarized in the following form (see Table 3).

It is difficult not to see here a striking symmetry – the basic regularities of the hyperbolic deceleration of the post-Big Bang universal increase in complexity turn out to be strikingly similar to the ones of the hyperbolic acceleration of the complexity growth observed on our planet for 4 billion years until the early 1970s.

Modis – Kurzweil global complexity growth acceleration pattern	**Panov global complexity growth acceleration pattern**
$y = 2.054{*}x^{-1.003}$ ($R^2 = 0.9989$), where y is the rate of the global (planetary) complexity growth; x is the time till the 21st century Singularity ($t^* = 2029$); $x = t^* - t$; $y = \frac{2.054}{(t^*-t)^{1.003}}$; $y = \frac{2.054}{t^*-t}$; $\boldsymbol{y} = \frac{C_1}{\boldsymbol{t^*-t}}$ $y = \frac{2.054}{2029-t}$.	$y = 1.886{*}x^{-1.01}$ ($R^2 = 0.9991$), where y is the rate of the global (planetary) complexity growth; x is the time till the 21st century Singularity ($t^* = 2027$); $x = t^* - t$; $y = \frac{1.886}{(t^*-t)^{1.01}}$; $y = \frac{1.886}{t^*-t}$; $\boldsymbol{y} = \frac{C_1}{\boldsymbol{t^*-t}}$ $y = \frac{1.886}{2027-t}$.
Universal complexity growth deceleration pattern	
$y = 0.549{*}x^{-0.998}$ ($R^2 = 0.999996$), where y is the rate of the universal complexity growth; x is the time since the Big Bang Singularity ($t^* = 13.8$ biillion BP); $x = t - t^*$; $y = \frac{0.549}{(t-t^*)^{0.998}}$; $y = \frac{0.549}{t-t^*}$; $\boldsymbol{y} = \frac{C_2}{\boldsymbol{t-t^*}}$ $y = \frac{0.549}{t - 13.8 \cdot 10^9\text{BCE}}$.	

Table 3. Comparison of the decelerating pattern of the universal (cosmic) evolutionary development evidenced in the Universe for a few billions of years since the Big Bang Singularity with the accelerating pattern of complexity growth traced on our planet detected in the series of Modis – Kurzweil and Panov

3.1 Relationship between the Cosmic Radiation Energy and Universal Complexity Growth Rate

Consider now the relationship between the radiation energy of the Universe and universal complexity growth rate = evolutionary megadevelopment rate (measured in phase transitions per year). We have used the following time series for our analysis, taking into account the following phases of the universal complexity growth and corresponding values of the radiation energy of the Universe (measured in eV).Below the same figure is presented with direct order of values along the x-axis (see Fig. 10).

$$\boldsymbol{y = C_4{*}E^2}, \qquad (9)$$

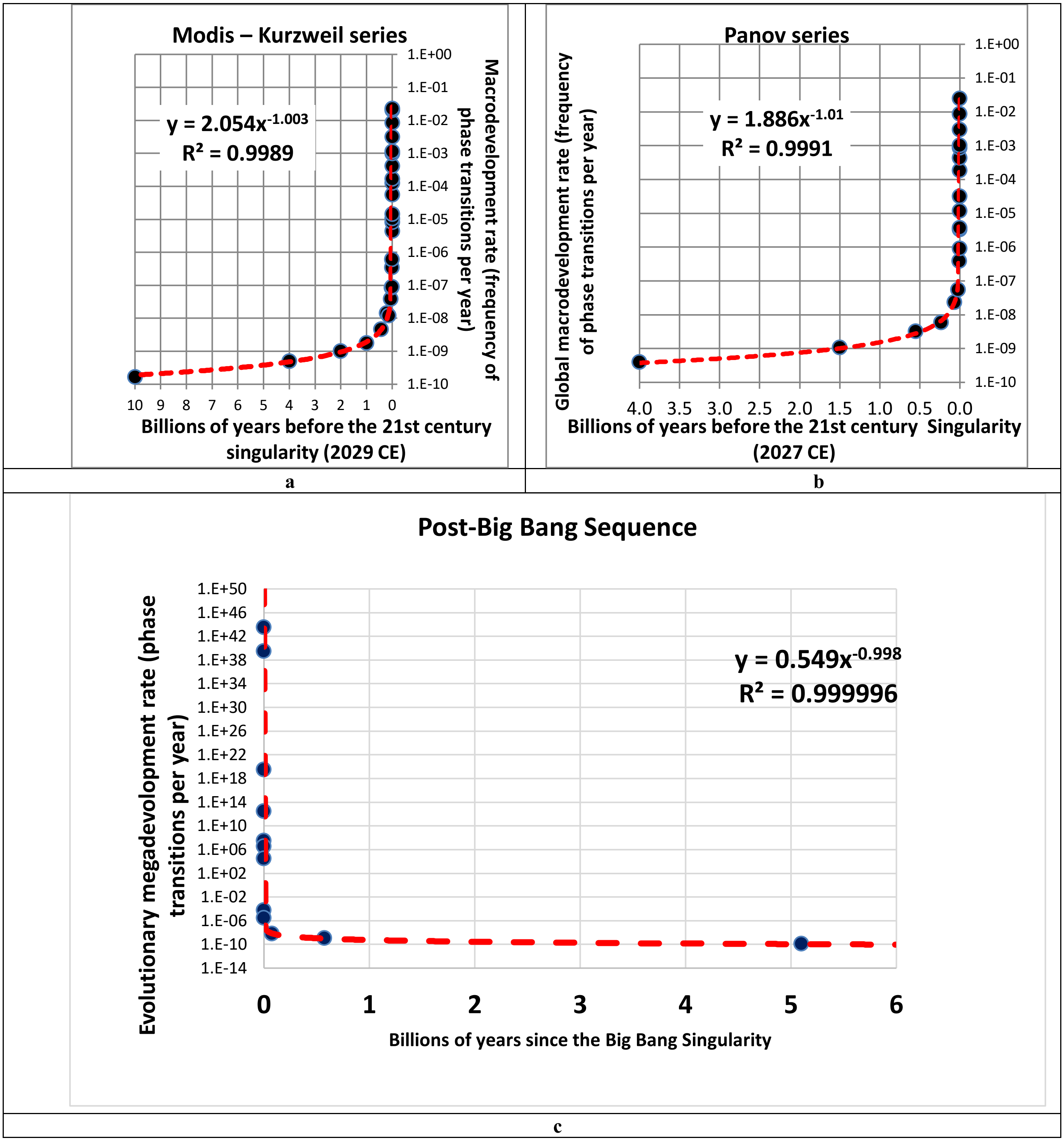

Fig. 9 Comparison of the decelerating pattern of the universal (cosmic) evolutionary development evidenced in the Universe for a few billions of years since the Big Bang Singularity (**c** above) with the accelerating pattern of complexity growth traced on our planet for four billion years since the emergence of life on the Earth detected in the series of Modis – Kurzweil (**a** above) and Panov (**b** above)

Decelerating universal (cosmic) evolutionary development	**Accelerating global (biosocial) evolutionary development**
$y = \frac{C_1}{t - t^*}$	$y = \frac{C_2}{t^* - t}$
Thus, the general formula of the deceleration of the universal (cosmic) complexity growth can be described as follows: • The rate of the universal (cosmic) complexity growth decreases when we move from the Singularity. • As the time since the Singularity increases *n* times, the universal (cosmic) complexity growth rate decreases the same *n* times. • Thus, if the time since the Singularity rises by a factor of 3, the speed of the universal (cosmic) complexity growth lessens 3 times; if the time since the Singularity increases 10 times, the universal (cosmic) complexity growth rate diminishes by a factor of 10, and so on.	Thus, the general formula of the acceleration of the global (biosocial) complexity growth can be described as follows: • The rate of the global complexity growth increases when we approach the Singularity. • As the time till the Singularity decreases *n* times, the global complexity growth rate increases the same *n* times. • Thus, if the time till the Singularity lessens by a factor of 3, the speed of the global complexity growth rises 3 times; if the time till the Singularity diminishes 10 times, the global complexity growth rate escalates by a factor of 10, and so on.

Table 4. Comparison of the decelerating pattern of the universal (cosmic) evolutionary development with the accelerating pattern of complexity growth (version 2)

3.2 Relationship between Cosmic Radiation Energy and Time Since the Big Bang Singularity

It can be easily shown analytically that if within the cosmic evolution the rate of the universal complexity growth *y* equals constant C_1 divided by the time since the Big Bang Singularity ($t - t^*$, or x)

$$y = \frac{C_1}{t-t^*} \tag{10}$$

and the rate of the universal complexity growth *y* is proportional to the radiation energy of the Universe *E* squared

$$y = C_4{*}E^2 \tag{11}$$

then the radiation energy/temperature of the Universe *E* should be proportional to some constant C_3 (= C_1/C_4) divided by a square root of the time since the Big Bang Singularity ($t - t^*$, or x):

$$\boldsymbol{E} = \frac{C_3}{\sqrt{t-t^*}} = C_3x^{-0.5} = C_3x^{-\frac{1}{2}} \tag{12}$$

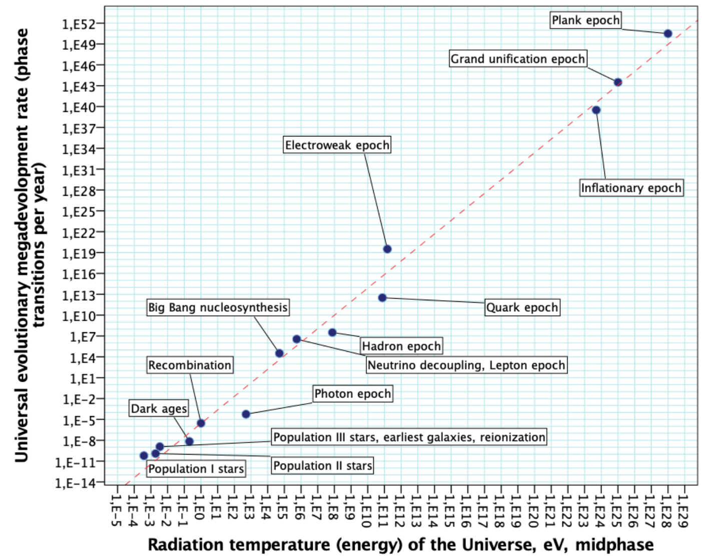

Fig. 10 Relationship between the radiation energy (temperature) of the Universe (eV) and universal evolutionary megadevelopment rate (phase transitions per year). Scatterplot of the phases of the growth of complexity in the Universe, with the fitted power-law regression line **(log-log scale, with direct order of values along the x-axis)**

Indeed, if $y = \frac{C_1}{t-t^*}$ and $y = C_4E^2$, then $C_4E^2 = \frac{C_1}{t-t^*}$. Thus, $E^2 = \frac{C_1}{C_4}\frac{1}{t-t^*}$. Hence, $E^2 = \frac{C_3}{t-t^*}$, where $C_3 = \frac{C_1}{C_4}$. So, finally we arrive at $E = \frac{C_3}{\sqrt{t-t^*}} = C_3x^{-0.5} = C_3x^{-\frac{1}{2}}$, where *E* is the radiation energy (temperature) of the Universe (eV), $x = t - t^*$ is time since the Big Bang Singularity and C_3 is a constant.

The analysis of the data presented above in Table 2 suggests that this is indeed the case. Our analysis has demonstrated that the relationship between time since the Big Bang Singularity (years) and radiation energy of the Universe (eV) can be quite accurately described by the following equation:

$$\boldsymbol{E} = \frac{C_3}{\sqrt{t-t^*}} = C_3x^{-0.5} = C_3x^{-\frac{1}{2}}, \tag{13}$$

where *E* is the radiation energy of the Universe (eV); *x* (or $t - t^*$) is the time since the Big Bang Singularity, and C_3 is a constant (see Figs. 11 and 12).

In fact, this relationship is well known in cosmology and may be derived from original Friedman's equations (see, e.g., Mukhanov, 2005: 72)[9].

[9] In fact, Eq. (12) describes quite accurately the relationship between the time since the Big Bang Singularity and the radiation energy (temperature) of the Universe for the radiation-dominated era of its history only, whereas for the matter-

This suggests that the post-Big Bang hyperbolic deceleration of the universal complexity growth was directly connected with the post-Big Bang hyperbolic deceleration of the cooling of the Universe described by Eqs. (10) and (12).

In fact, this suggests that the above detected hyperbolic pattern of deceleration of the post-Big-Bang universal complexity growth rate is not just an artefact of some dubious numerological exercise, but rather reflects a well-established scientifically pattern of the hyperbolical slowdown of the speed of the cooling of the Universe.

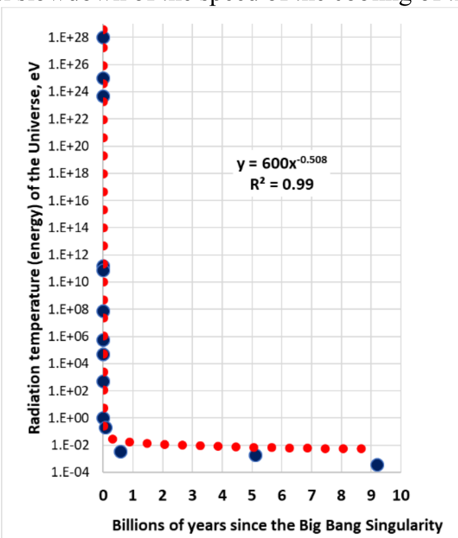

Fig. 11 Correlation between the time since the Big Bang Singularity (years) and radiation energy (temperature) of the Universe (eV). Scatterplot of the phases of the growth of complexity in the Universe, with the fitted power-law regression line **(with a logarithmic scale for the Y-axis)**

After the Big Bang Singularity, the growth of complexity in the Universe was very tightly connected with its cooling. It was this cooling that allowed the formation in the Universe of more and more complex entities – quarks, then hadrons, then atomic nuclei, then atoms, then molecules (see, e.g., Baumann, 2022; Gorbunov & Rubakov, 2018; Grinin, 2013; Hawking, 2009; LePoire, 2016; Mukhanov, 2005; Ryden, 2017). At the very beginning the cooling of the Universe proceeded very fast, and the complexity in the Universe grew extremely fast (with a few phase transitions just within the first second after the Big Bang Singularity). Then the cooling of the Universe slowed down, which caused the slowing down of the growth of complexity in the Universe.

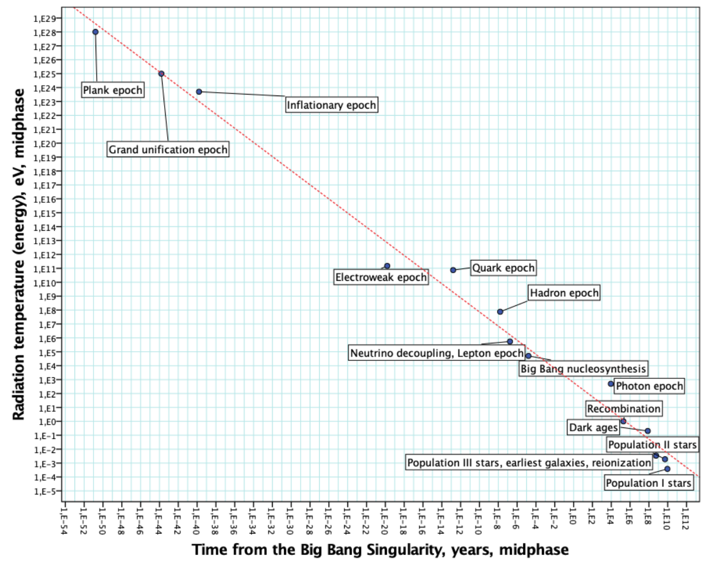

Fig. 12 Correlation between the time since the Big Bang Singularity (years) and radiation temperature (energy) of the Universe (eV). Scatterplot of the phases of the growth of complexity in the Universe, with the fitted power-law regression line **(log-log scale)**

As we have seen, the slowing down of the cooling of the Universe followed a hyperbolic pattern, and it does not appear to be of any surprise that the hyperbolic slowdown of the cooling of the Universe after the Big Bang Singularity caused a hyperbolic slowdown of the universal complexity growth rate.[10]

4 Relationship between Energy and Complexity Growth Rate in Global Development

Consider now the relationship between time till the 21st century singularity (years) and world energy production (TWy) estimated by John Holdren (1991; see Fig. 13):

dominated era it is much better described by another equation (with -2/3 rather than -1/2 as the exponent):

$$E = C_3 x^{-\frac{2}{3}} \text{ (see, e.g., Mukhanov, 2005: 124).} \quad (14)$$

Note that this point explains why our mathematical analysis of the empirical estimates above (see Table 6 and Fig. 11) has produced a version of Eq. (14) with the exponent higher than 0.5. This is due to the fact that our analysis included a number of data points from the matter-dominated era. However, as the number of data points from the energy-dominated era exceeded the number of ones from the matter-dominated era, the value of the exponent turned out to be closer to 0.5 rather than 0.67.

[10] But it may well be said the other way around: at the beginning the concentration of the energy in the Universe was extremely high, which resulted in the extremely high rate of complexity growth, whereas the subsequent hyperbolic decline of the universal energy concentration resulted in the hyperbolic decrease of the rate of the growth of complexity in the post-Big-Bang Universe (e.g., LePoire, 2016: 229–230).

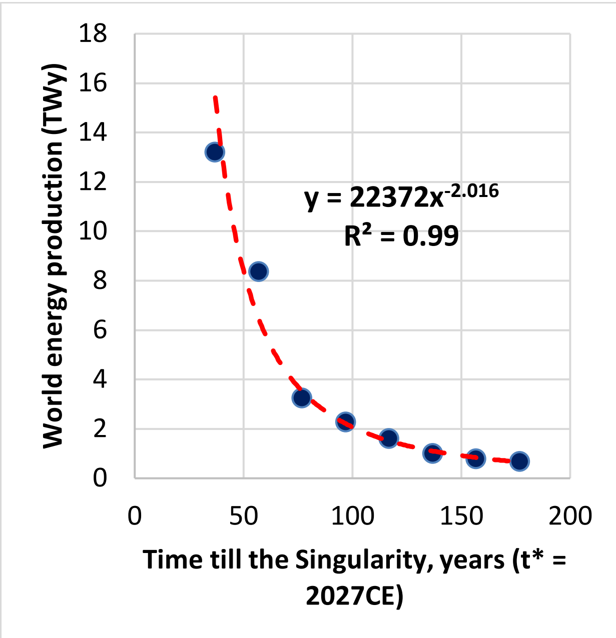

Fig. 13 Relationship between the time till the Singularity, years (t* = 2027CE) and the world energy production (TWy). *Data source*: Holdern, 1991: 245.

As we see, for the pattern of global hyperbolic acceleration we find a quadratic relationship between the energy production and the time till the singularity inversed to the one we saw with respect to the post-Big-Bang universal deceleration:

$$\boldsymbol{E} = \frac{C_6}{(\boldsymbol{t}^{*} - \boldsymbol{t})^2}, \tag{15}$$

where E is the world energy production, $t^* - t$ is the time till the Singularity, and C_6 is a constant.

Correspondingly, the relationship between world energy production (E, TWy) and global complexity growth rate (y, phase transitions per year) is described by the following equation:

$$\boldsymbol{y} = \boldsymbol{C_5}\sqrt{\boldsymbol{E}} \tag{16}$$

Thus, the growth of the world energy production 4 times only leads to a twofold increase in the global complexity growth rate; whereas in order for the global complexity growth to increase 4 times, the world energy production should grow by a factor of 16.

Note that this pattern is symmetrically opposite to the one we confronted above dealing with the post-Big-Bang deceleration of the universal complexity growth (see Eq. (12)), when the decrease of the universal radiation energy 4 times led to the decrease of universal complexity growth rate by a factor of 16.

Table 5 below summarizes the general mathematical description of decelerating universal (cosmic) evolutionary development:

Relationship between time since the Big Bang Singularity (***t-t****, years) and universal complexity growth rate (***y***, phase transitions per year)	$\boldsymbol{y} = \frac{\boldsymbol{C_1}}{\boldsymbol{t} - \boldsymbol{t}^*}$
Relationship between time since the Big Bang Singularity (***t-t****, years) and radiation energy (temperature) of the Universe (***E***, eV)	$\boldsymbol{E} = \frac{C_3}{\sqrt{\boldsymbol{t} - \boldsymbol{t}^*}}$
Relationship between radiation energy (temperature) of the Universe (***E***, eV) and universal complexity growth rate (***y***, phase transitions per year)	$y = C_4 * E^2$

Table 5 Summary general mathematical description of decelerating universal (cosmic) complexity growth

5 Complexity Growth Comparison of Cosmic Deceleration and Global Acceleration

A general mathematical comparison between decelerating universal (cosmic) evolutionary development and accelerating global (biosocial) evolutionary development is presented below at Table 6.

As we see, the correlations between energy and decelerating growth of universal complexity display a striking inversed symmetry in comparison with accelerating global evolutionary development.

In the cosmic history, the rate of the universal complexity growth was proportional to the radiation energy of the Universe squared. In the global history, the rate of the global complexity growth was proportional to the square root of the world energy production (see Table 6, Row 2).

In the cosmic history, the moving from the Big Bang Singularity (Singularity$_1$) by n times was accompanied by the decrease of the radiation energy of the Universe by $\sqrt{n}$ times. Thus, the increase in the time since Singularity$_1$ by a factor of 4 was associated with the drop in the radiation energy of the Universe by a factor of 2. On the other hand, in the global history the moving toward the 21st century Singularity (Singularity$_2$) by n times was associated with growth of the world energy production by n^2 times. Thus, the decrease in the time till Singularity$_2$ by a factor of 4 was associated with the increase in the world energy production by a factor of 16 (see Table 6, Row 2).

Finally, Row 1 of Table 6 demonstrates a perfect symmetry already discussed above: (1) the rate of the universal (cosmic) complexity growth decreases when we move from Singularity$_1$, whereas the rate of the global complexity growth increases when we approach Singularity$_2$; (2) more specifically, as the time since Singularity$_1$ increases n times, the universal (cosmic) complexity growth rate decreases the same n times, whereas when the time till Singularity$_2$ decreases n times, the global complexity growth rate increases the same n times; (3) even more specifically, if the time since Singularity$_1$ rises by a factor of 3, the speed of the universal (cosmic) complexity growth lessens 3 times; if the time since

Singularity_1 increases 10 times, the universal (cosmic) complexity growth rate diminishes by a factor of 10, and so on. On the other hand, if the time till Singularity_2 lessens by a factor of 3, the speed of the global complexity growth rises 3 times; if the time till Singularity_2 diminishes 10 times, the global complexity growth rate escalates by a factor of 10, and so on.

Relationship #	**Accelerating global (biosocial) evolutionary development**	**Accelerating global (biosocial) evolutionary development**
1)	Relationship between time since the Big Bang Singularity (***t-t****, years) and universal complexity growth rate (***y***, phase transitions per year) $$y = \frac{C_1}{t - t^*}$$	Relationship between time till the 21st century singularity (***t*-t***, years) and global (biosocial) complexity growth rate (***y***, phase transitions per year) $$y = \frac{C_2}{t^* - t}$$
2)	Relationship between radiation energy (temperature) of the Universe (***E***, eV) and universal evolutionary megadevolopment rate (***y***, phase transitions per year) $$y = C_4 * E^2$$	Relationship between world energy production (***E***, TWy) and global (biosocial) evolutionary megadevelopment rate (***y***, phase transitions per year) $$y = C_5\sqrt{E}$$
3)	Relationship between time since the Big Bang Singularity (***t-t****, years) and radiation energy (temperature) of the Universe (***E***, eV) $$E = \frac{C_3}{\sqrt{t - t^*}}$$	Relationship between time till the 21st century singularity (***t*-t***, years) and world energy production (***E***, TWy) $E = \frac{C_6}{(t^* - t)^2}$

Table 6 General mathematical comparison between decelerating universal (cosmic) evolutionary development and accelerating global (biosocial) evolutionary development

6 Concluding remarks

Of course, this paper poses more questions than it answers. The most important of those questions seems to be – why do the basic regularities of the hyperbolic deceleration of the post-Big Bang universal increase in complexity turn out to be so strikingly similar to the ones of the global hyperbolic acceleration of the complexity growth when their mechanisms seem to be so different?

On the one hand, it has been shown that the global hyperbolic acceleration pattern of the last 4 billion years appears to have been produced endogenously by the second order positive feedback between the complexity of the global sociobiological system and the rate of its complexity growth: the more complex the global biosocial system, the less time it takes it to make the next complexity jump – thus, the more complex the global system was, the faster its complexity grew. It has been shown that when written mathematically, such a feedback produces precisely a hyperbolic acceleration effect (see, e.g., von Foerster et al., 1960; Taagepera, 1976, 1979; Kremer, 1993; Kurzweil, 2001; Tsirel, 2004; Korotayev, 2005, 2006a, 2006b, 2007, 2013, 2018, 2020b; Korotayev & Khaltourina, 2006; Korotayev et al., 2006a, 2006b, 2015, 2016; Markov & Korotayev, 2007, 2008; Korotayev & S. Malkov, 2012; Grinin et al., 2013; Korotayev & A. Malkov, 2016; Korotayev & Markov, 2014, 2015; LePoire, 2014, 2015a, 2015b, 2016, 2020a, 2020b).

On the other hand, as we have seen above, the hyperbolic deceleration of the post-Big-Bang universal complexity growth rate appears to have been produced exogenously by the post-Big-Bang hyperbolic deceleration of the cooling of the Universe: the slower this cooling proceeded, the slower the universal complexity grew – thus, the post-Big-Bang hyperbolic deceleration of the cooling of the Universe resulted in the hyperbolic deceleration of the post-Big-Bang universal complexity growth rate.

Yet, those apparently so different mechanisms appear to have produced such strikingly similar patterns of hyperbolic deceleration / acceleration.

Of course, this point needs further investigations.

7 References

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